

# Stochastic resonance induced by fluctuation in liquid membrane oscillator without input signals

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## Abstract

We investigated numerically the dynamic behavior of the oil/water liquid membrane, which is a promising model for excitable bio-membrane. When we use noise to modulate the parameters in simulation, noise-induced coherent oscillation is observed. With the increment of the noise intensity, the coherence of noise-induced oscillation can go through a maximum, which indicating the occurrence of stochastic resonance (SR) without input signals. We compared the SR effects under the condition that noise is added to different control parameters. When noise was added to both of the parameters, a complicated SR-like phenomenon was observed. The interaction of coherent SRs induced by two independent noises is discussed. The possibly constructive role of noise in some sensory cells is discussed also. © 2001 Elsevier Science B.V. All rights reserved.

**Keywords:** Stochastic resonance; Liquid membrane; Oscillation

## 1. Introduction

In living organisms for maintaining life, the process of bio-membranes is a very important part. Many scientists have paid more attention to the dynamic process of excitable cell membranes. The oscillation of membrane current and/or po-

tential is closely related to the electrical excitability in living organism [1]. This subject has been a focus of research in life science and has been investigated extensively by many authors using bio-membranes of artificial membranes [2,3]. These oscillations are expected to offer fundamental information useful in elucidating the oscillation processes at bio-membrane in living organisms [2,3] because of some similar characteristics, such as high sensitivity and selectivity [4,5].

In the past decades, many excitable artificial

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liquid membranes have been studied. Among them, the oil/water liquid membrane, which was first reported by Dupeyrat and Nakache in 1978 [6], has been considered as the most simple and effective model for excitable bio-membrane [2,3]. It was been extensively studied experimentally and theoretically [7–11]. Recently, the experiments of the oscillation of the oil/water membrane have been greatly improved [13,14]. Some applications of it have been studied recently [15]. However, the effects of fluctuations were not taken into account in the previous work.

It is well known that interplay between deterministic non-linear dynamics and noise can lead to non-trivial phenomena such as noise-induced transition [16] and stochastic resonance (SR) [17]. The concept of SR was originally put forward by Benzi et al. [18]. Then Fauve and Heslot first verified it experimentally in a noise-driven electronic circuit known as the Schmitt trigger [19]. Many scientists have paid considerable attention to these counter intuitive phenomena in which noise plays a constructive role rather than as a negative one [20], such as in chemical reactions etc. [21–25].

Recently, it has been shown that non-linear systems in the presence of noise can also display SR-like behavior, even without external signal [26–29]. These phenomena can be called autonomous SR [26], coherence resonance [27–29], or internal SR, i.e. the coherence of noise-induced motion of a non-linear system could reach a maximum with the increment of noisy intensity.

One of the major motivations of SR studies is its application in biology, especially in excitable neuronal system [31]. It has been shown both experimentally [32–34] and theoretically [17] that the ability of sensory neurons to detect weak input signals can be enhanced by adding noise to the system [35]. We have reported the SR phenomenon in oil/water oscillator model by Yoshikawa with external signals [37]. In this paper, we investigated the dynamics of the oil/water liquid membrane oscillator modulated by noise on different parameter without external signals. The deterministic oscillation induced by noise was observed. Furthermore, the interaction of the two independent modulations on two parameters

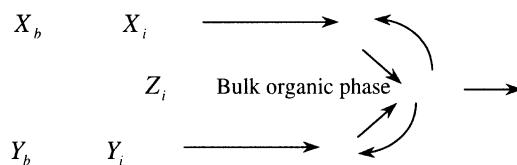
was also studied. The possible functional role of noise in this bio-membrane model was discussed. The results show that biological systems maybe take advantage of noise through internal SR to maintain their coherent order.

## 2. Dynamic model of liquid membrane oscillator

So many possible interpretations and models have been proposed in the last decades, including of the hydrodynamics proposed by Dupeyrat and Nakache in 1978 [6] and the two models proposed by Yoshikawa and his co-works in 1983 [7] and 1988 [9]. T. Kawakubo also proposed one model based on diffusion process and reaction process at the interface in 1988 [12]. Among these, the second model proposed by Yoshikawa and his co-workers [9,11] is the most complete and successful. The whole process can be shown as following:

Here  $X_b$  and  $Y_b$  stand for the concentration of the surfactant and the co-operative species (usually alcohol) in the bulk aqueous phase and  $X_i$ ,  $Y_i$  and  $Z_i$  are the concentration of the surfactant, the co-operative species and the complex of  $X$  and  $Y$ , respectively, near the oil/water interface. The subscripts b and i, represent the sites at the o/w interface and the bulk water phase, respectively. From bulk aqueous solution phase and aggregate at the interface to form the complex  $Z$ . When  $Z$  gathers up to a critical coverage, it is abruptly transferred across the interface into the organic phase. The periodic formation and disruption of the monolayer of the complex  $Z$  are responsible for the oscillations. The related governing dynamic equations read:

$$\frac{dX_i}{dt} = D_x(X_b - X_i) - K_1Z_i \quad (1)$$



$$\frac{dY_i}{dt} = D_y(Y_b - Y_i) - K_2 Z_i \quad (2)$$

$$\frac{dZ_i}{dt} = K_3(X_i + Y_i) - K_4 G(Z_i) \quad (3)$$

On the right-hand side of Eq. (1) and Eq. (2), the first term denotes the diffusion only for the case where a linear concentration gradient exists, the second term corresponds to the negative feedback of the aggregate or complex between the surfactant ( $X_i$ ) and the co-operative species ( $Y_i$ ) molecules. As the surface pressure of the monolayer is expected to be proportional to the two-dimensional density of  $X_i$  plus  $Y_i$ , the synergistic effect of  $X_i$  and  $Y_i$  is assumed to be simple addition in Eq. (3). The function  $G(Z_i)$  is an  $N$ -shaped one, which enables the system to be excitable. It is related to the statistical behavior of the surfactant at the oil/water interface and based on both van der Waals-type attraction between the molecules and Debye–Huckel theory. Detailed discussion of the physical meaning of this non-linearity can be found in ref. [9]. Here, we make:

$$G(Z) = Z^3 + K_5 Z^2 + K_6 Z + K_7 \quad [32] \quad (4)$$

When we choose the suitable parameters, Eq. (1) exhibits oscillation, shown in Fig. 1.

### 3. Results

If we choose  $K_3 = K_3^0$  and  $X_b = X_b^0$ , keeping the other parameters the same as shown in Fig. 1, the system will remain stable, with no oscillation (Fig. 2a). We will random modulated  $X_b$  and  $K_3$  by stochastic force to investigate numerically the dynamic behavior of this system. (Here,  $K_3^0 = 21.0$ , it is a constant.  $X_b^0 = 2.475$ , it is the constant concentration of  $X_b$  at the focus.)

#### 3.1. Random modulation of $X_b$

First we modulate the parameter  $X_b$ , let  $X_b = X_b^0 + \beta\Gamma(t)$ , stands for a Gaussian white noise with zero mean  $\langle\Gamma(t)\rangle = 0$  and unit variance

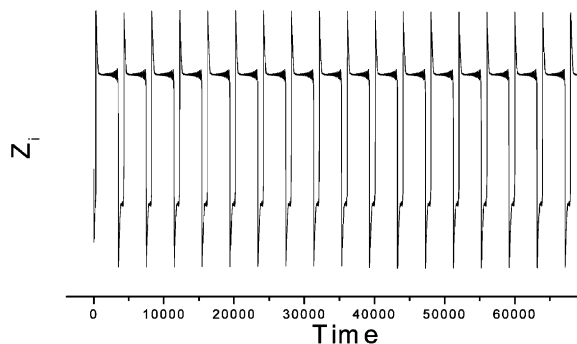


Fig. 1. Computer simulation of the oscillation of  $Z_i$  parameters:  $D_x = 2.0$ ;  $D_y = 1.0$ ;  $Y_b = 3.0$ ;  $X_b = 2.475$ ;  $K_1 = 1.4$ ;  $K_2 = 1.4$ ;  $K_3 = 21.1$ ;  $K_4 = 5.0$ ;  $K_5 = -7.0$ ;  $K_6 = 3.30$ ; and  $K_7 = 20.0$ . (These parameters remain unchanged except for special setting).

$\langle\Gamma(t)\Gamma(t')\rangle = 2\delta(t-t')$ ,  $\beta$  is the noise intensity. The Kinetic equations including Eq. (4), were integrated numerically by using the fourth-order Runge-Kutta method ( $\Delta t = 0.005$ , total time  $5 \times 10^3$ , total number of data 100 000). To quantify the SR without external signals, the time series of  $Z_i$  was analyzed by the Fourier spectra.

Fig. 2b,c,d show the time series of  $Z_i$  when  $\beta$  is, respectively, 0.008, 0.036 and 0.05. One can see the occurrence of the noise-induced oscillation. The Fig. 3a shows the Fourier power spectrum, where one can see that a clear peak appears (it is obtained by averaging over 50 independent runs). The Fig. 3b shows the SNR to  $\beta$ , one can see that with the increment of noise intensity the signal-to-noise ratio (SNR) [41] go through a peak, which indicating the occurrence of the SR.

#### 3.2. Random modulation of $K_3$

Then we used noise to modulate  $K_3$  in our simulation, let  $K_3 = K_3^0 + \beta\Gamma(t)$ , where  $\Gamma(t)$  is again the Gaussian white noise. There is also a peak on Fig. 4a, which represents the Fourier power spectrum when  $\beta = 0.003$ . Fig. 4b shows the SNR vs.  $\beta$ , the occurrence of NICO and SR is also observed. Comparing Fig. 4b with Fig. 3b, one can see that the intensity of the noise added on  $K_3$  to make the SNR reach the maximum is smaller than that added on the  $X_b$ . The coherent

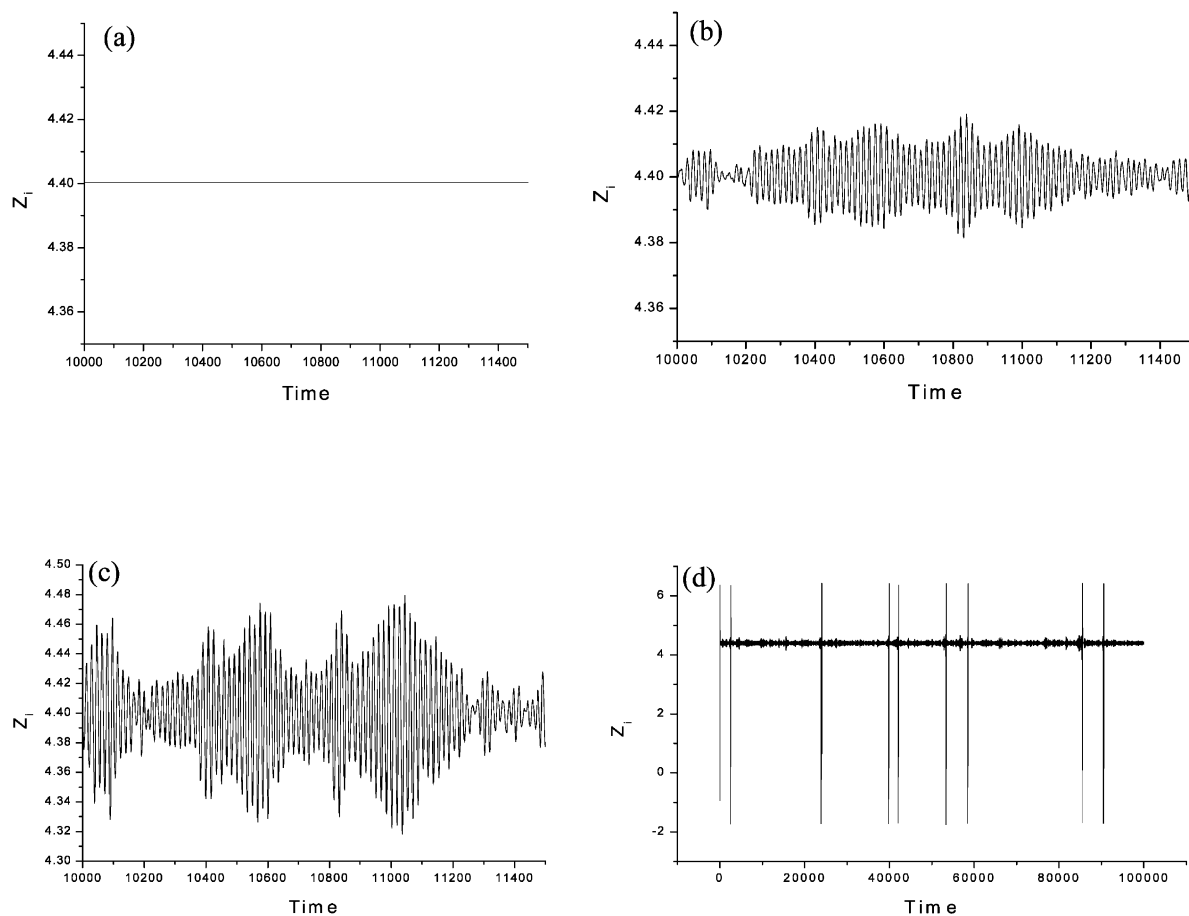


Fig. 2. Time series of  $Z_i$  with variation of the noise intensity. (a)  $\beta = 0$ , (b)  $\beta = 0.008$ , (c)  $\beta = 0.036$  and (d)  $\beta = 0.05$  (here  $K_3 = K_3^0$ , other parameters are the same as in Fig. 1).

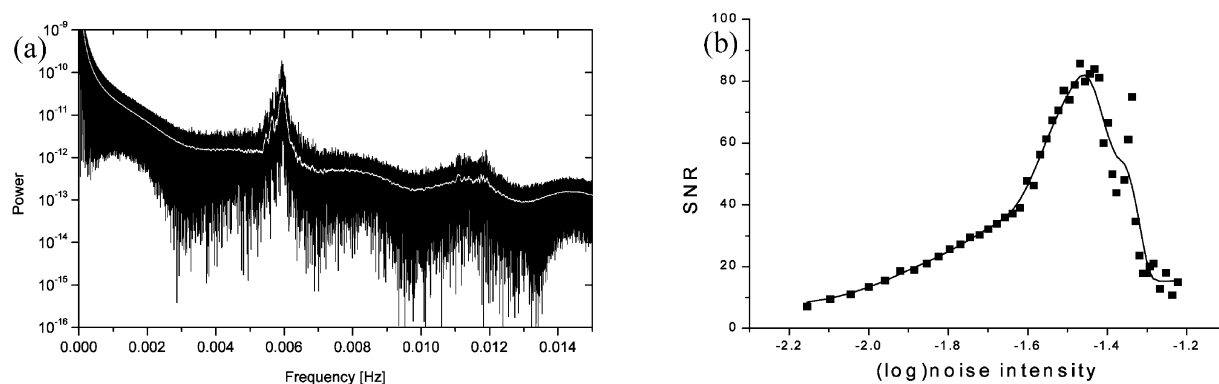


Fig. 3. (a) Fourier spectra of the time series of the  $Z_i$  when  $\beta = 0.036$ . (Noise added on  $X_b$ ) and (b) the dependency of the signal-to-noise (SNR) on noise intensity  $\beta$ .

oscillation of the system is more sensitive to the stochastic force on  $K_3$  than that on  $X_b$ .

### 3.3. Random modulation of $K_3$ and $X_b$ at the same time

The fluctuation on  $K_3$  mainly comes from the stochastic force inside the system and the fluctuation of  $X_b$  mainly comes from the influence of the environment. In fact, these two types of fluctuation always must all exist in bio-membrane systems at same time, so it is important to study the interaction of these two types fluctuation. When we let  $X_b = X_b^0 + \beta_1 \Gamma(t)$ ,  $K_3 = K_3^0 + \beta_2 \Gamma(t)$ , (here  $\beta_1$  and  $\beta_2$  are the intensity of the noise,  $\Gamma(t)$  and  $\Gamma(t')$  are generated by different noise seeds). Noise-induced oscillation can also be observed. Fig. 5a shows us the three-dimensional-wire surface of SNR- $\beta_1$ - $\beta_2$  and Fig. 5b shows us the contour of the SNR- $\beta_1$ - $\beta_2$ .

## 4. Discussion

In the present paper, we have studied the noisy dynamic behavior of the oil/water liquid membrane system, which has been investigated by Yoshikawa and co-workers extensively with many experiments. It is thought of as the most successful artificial membrane model to simulate excitable bio-membranes. The importance of this

model in the present context is related to the fact that it reproduces qualitative behavior of some sensory cells. So the SR induced by the fluctuation on two different parameters is taken into account. And we also investigated the SR when these two stochastic forces are added on at the same time. The following conclusions can be made from our study:

1. Noise-induced oscillations are observed when the stochastic force is added on to different parameters of the system. On each condition, the SR is observed, that means that the suitable intensity of the noise will enhance the noise-induced oscillation strength to a maximum. For optimal noise intensity, the dynamics of system is most coherent.
2. When we used the noise to modulated two different parameters of the oil/water liquid system model, a new SR-like phenomenon is observed. It is interesting that one can clearly see that there are two peaks and one saddle-point in the surface figure (Fig. 5). That means there are two groups of suitable values of  $\beta_1$  and  $\beta_2$  to make the system come to maximum coherence and there is one group of suitable values of  $\beta_1$  and  $\beta_2$  to make the SNR come to a saddle-point. We think all these come from the interaction of these two stochastic forces. That means that the noise from different sources can interact with each other in

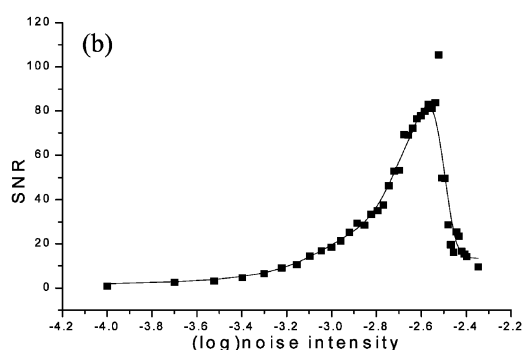
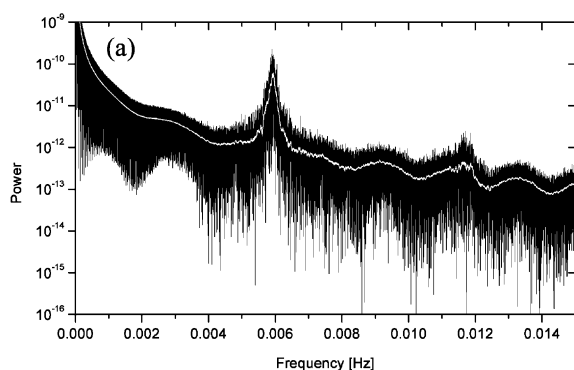


Fig. 4. (a) Fourier spectra of the time series of the  $Z_i$  when  $\beta = 0.003$ . (Noise added on  $K_3$ ) and (b) the dependency of the signal-to-noise (SNR) on noise intensity  $\beta$ .

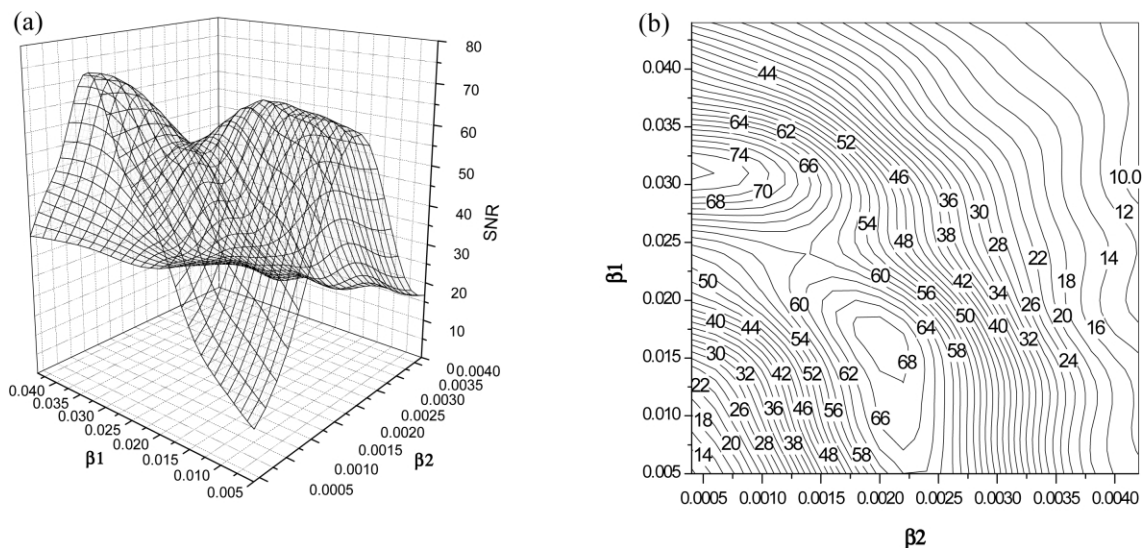


Fig. 5. (a) The surface plot of the signals-to-noise (SNR) vs. noise intensity  $\beta_1$  and  $\beta_2$  and (b) the contour plot of the signals-to-noise (SNR) vs. noise intensity  $\beta_1$  and  $\beta_2$ .

the non-linear system and it will affect the coherent dynamics behavior.

Then let us have a qualitative discussion of the mechanism underlying the noise-induced relaxation oscillation in the oil/water membrane. The reason for it was that noise let the system randomly visit the oscillatory region. When the system parameters are close to the regime of self-oscillations then noise can initialize the oscillation behavior. For a weak noise the system spends a long time in basin of attraction of the fixed point and rarely makes a transition to the limit cycles. With the increment of noise intensity, the system will be more and more coherent, so the SNR increases also. However, strong noise dramatically influences the induced-oscillation and leads to the amplitude and phase fluctuation and, therefore, destroys the coherence of noise-induced oscillation. Recent experimental observations of noise-initiated and sustained long-range coherent waves of calcium ions in cultured brain tissue [38] and noise-supported wave propagation in the photosensitive BZ reaction indicated a similar underlying dynamical process [30]. Until now, no fundamental theory of internal SR ex-

isted. Further theoretical work will be of great help for future research.

The oil/water membrane model presented in this study involves two independent bifurcation parameters,  $X_b$  (concentration) and  $K_3$  (synergetic effect constant), which will represent the different physical roles in the real oil/water systems. Two-parameter SR or, in general, multi-parameter SR may widely be seen in biological and natural systems [32–34]. As a promising model for excitable bio-membranes, it is import to study the internal multi-parameter SR. Maybe it will provide further insight into the behavior sensory membrane.  $K_3$  stands the synergetic effect  $X_i$  and  $Y_i$  to form  $Z_i$ , its fluctuation can come from the spatial inhomogeneity [39] of the species near the interface. When Sagues et al. studied stochastic dynamics due to spatial inhomogeneity in a chemical system, they incorporated noise terms through the rate constants [40].  $X_b$  stands the concentration at the bulk water phase, it is easily affected by the environment, such as temperature etc. Therefore, it is possible that SR is a very common phenomenon in this kind of system and might also be a potential functional mechanism in which the coherent oscillation can be detected

effectively. Our study shows that the more noise exists, the more effectively systems use the mechanism of SR.

Bio-oscillation is one of the most important properties of living organisms. Some of these oscillations maybe a kind respond to stimulates and the mode of oscillations is also important. In fact, noise induced by internal or external fluctuations inevitably affects these bio-oscillation processes. Because of their same dynamic behaviors [3,7–11], the oil/water liquid membrane oscillator is considered to model the membranes of some sensory cells, such as olfactory and gustatory cells. So studying the effect of noise on this model is worth attention. Because there various kinds of noise resource in nature which affect different parameters of the bio-system, we also investigate the coherent SRs when induced noises were added on two independent parameters. Our result shows that internal SR may be a very common phenomenon in these systems. SR has been proposed as a means for improving signal detection and transmission in a considerable variety of systems [20,36], especially in some natural systems [31–35], such as sensory neurons. Internal SR implies that environmental fluctuations can induce internal intrinsic order of a non-linear system. Bio-systems maybe take advantages of noise through SR to maintain their internal order. Further experimental and theoretical work will be helpful to the study of the role of noise in biological systems.

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- [41] Here, the word SNR is used to express the coherence of noise-induced oscillations. It is measured by the relative height of the peak  $h$ . (Notice that a more appropriate measure of the strength of the NICO might be the  $h/\Delta\omega$ , here  $\Delta\omega$  is the width of the peak at the height of  $h/2$ , but in the present work, we only consider the qualitative behavior and  $h$  is acceptable.)